Monetary Policy, Interest Rate Rules, and Inflation Targeting: Some Basic Equivalences

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Monetary policy in small open economies is typically cast as a choice between an exchange rate anchor (fixed or predetermined exchange rates) and a money anchor (floating exchange rates). Under such regimes, the growth rate of the nominal anchor is set according to the desired long-run inflation rate. After undergoing a not necessarily painless adjustment process, the economy would eventually reach the long-run inflation rate.

In practice, however, policymakers have certainly not restricted themselves to such a limited menu of policy instruments. Pure floating rates are, at best, rare, as policymakers typically intervene in foreign exchange markets to smooth exchange rate fluctuations or achieve some international reserves target. While predetermined exchange rates are more common, it is still the case that, more often than not, policymakers adjust the devaluation rate in response to changes in the domestic and external environment or engage in real exchange rate targeting.[1]

At an even more fundamental level, policymakers increasingly view short-term nominal interest rates as the main nominal anchor. In developed countries, short-term interest rates are, by and large, the most common policy instrument (see Batten and others, 1990). The most prominent example is, of course, the United States: the Federal Reserve conducts monetary policy by setting the federal funds rate (the interest rate at which commercial banks borrow overnight). When

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1. See, for instance, Bruno (1993); Calvo, Reinhart, and Végh (1995); Lahiri (1997).

Indexation, Inflation, and Monetary Policy, edited by Fernando Lefort and Klaus Schmidt-Hebbel, Santiago, Chile.

inflation raises its ugly head, the Federal Reserve engages in a gradual tightening of monetary policy by raising the federal funds rate.

In developing countries, short-term interest rates have also played a key role in the conduct of monetary policy. In the mid-1980s, for instance, countries such as Argentina and Brazil supported repeated stabilization attempts by hiking interest rates on short-term government debt to increase demand for domestic assets and thus prevent speculative attacks against the domestic currency (see Calvo and Végh, 1995). In the case of Mexico, analysts point to the monetary authorities' reluctance to raise interest rates as a key factor in triggering the December 1994 crisis. In the wake of Thailand's decision to abandon its fixed exchange rate system in July 1997, Brazil and Hong Kong repeatedly raised interest rates to thwart currency speculators.

With the advent of indexed government debt, some countries—Chile being the most notable example—have actually used the interest rate on such instruments (that is, a real interest rate) as the main policy instrument (see, for example, Corbo and Fischer, 1994). Even when policymakers do not set a real interest rate, a real interest rate target seems to be very much on their minds. Reinhart (1993), for instance, argues that the Federal Reserve may be viewed as aiming at setting a real interest rate that is consistent with full-employment output. In fact, the level of the federal funds rate in real terms is often an important consideration in whether to raise further interest rates.

In practice, the use of interest rates as the main policy instrument has often taken place in conjunction with some inflation target. In other words, policymakers set an inflation target—explicitly or implicitly—and change interest rates with the aim of achieving such a target. When inflation targets are explicitly announced, such policies have been referred to as inflation targeting (see Leiderman and Svensson, 1995; Masson, Savastano, and Sharma, 1997). Inflation targeting has been implemented in industrial countries such as Australia, Canada, Finland, New Zealand, Spain, Sweden, and the United Kingdom. Although, in principle, a myriad of policy instruments could be used to achieve a given inflation target, in practice short-term interest rates have served as the main policy instrument. Hence, the most common manifestation of inflation targeting appears to be an inflation target accompanied by some explicit or implicit interest rate rule that is designed so as to achieve the target. Among developing countries, Chile seems to come the closest to using an inflation targeting scheme (see Masson, Savastano, and Sharma, 1997).

At an analytical level, the use of nominal interest rates as a policy instrument has raised some important questions regarding its impact on prices and inflation. Nominal interest rate targeting may lead to an indeterminate price level under flexible prices (Sargent and Wallace, 1975) or indeterminate inflation rate under sticky prices (Calvo, 1983). Indeterminacy problems can be avoided by explicitly introducing a government budget constraint (Auernheimer and Contreras, 1993); by designing appropriate interest rate rules (see, for instance, Reinhart, 1992); or by letting policymakers set the interest rate on liquid bonds issued by the government (Calvo and Végh, 1995, 1996). Policy rules that rely on real interest rates may also easily lead to various indeterminacies (Reinhart, 1993). The use of the real interest rate as a policy instrument has received particular attention in the case of Chile (see, for example, Rojas, 1993; Corbo and Fischer, 1994; Mendoza and Fernández, 1994). Although some important insights have followed from this large literature, the profession is still far from reaching any sort of consensus on the comparative advantages of different instruments and policy rules.

This paper starts from the premise that in order to assess different policy rules, it is useful to establish some basic equivalences among policy rules. Equivalent policy rules are defined as rules that yield exactly the same dynamics in response to, say, a long-term reduction in the inflation rate. In a formal sense, finding equivalent policy rules is a trivial exercise. Consider the exercise of a permanent reduction in the rate of monetary growth in a closed-economy model. This exercise will generate as an outcome some equilibrium path for the nominal interest rate. Clearly, if policymakers could announce a credible inflation target (which would anchor the long-run inflation rate) and set that same path for the nominal interest rate, the monetary growth rate would endogenously fall on impact and remain at that lower level forever. These two policy rules—the fixed money growth rule and the interest rate rule—would therefore be equivalent.

In the real policy world, however, this formal equivalence will be relevant only if the endogenous path of the nominal interest rate in the fixed money growth rule is a linear function of observable variables, such as the deviation of the actual inflation rate from its long-run steady state. In that case, an interest rate rule that sets the interest rate according to the deviation of the actual inflation rate from an inflation target would achieve the same results. Otherwise, policymakers would not be able to implement the interest rate rule that replicates the fixed money growth rule.

This paper illustrates, in the context of a closed-economy model, the existence of some basic equivalences among policy rules (that is, policy rules that can be implemented in practice). Inflation is assumed to be a predetermined variable, reflecting the highly inertial nature of inflation in many countries with moderate inflation. The paper starts by comparing the use of a money anchor, a nominal interest rate anchor, and a real interest rate anchor, whereby policymakers set a fixed level of the corresponding policy instrument (that is, there is no feedback mechanism). It is shown that only a money anchor (that is, a *k*-percent money growth rule) yields a well-behaved system. A fixed level of either the nominal interest rate or the real interest rate leads to unstable dynamics. This first result, which is here derived for the case of sticky inflation, is thus in the spirit of the indeterminacies found in the literature. In theory, then, there is no substitute for a *k*-percent rule. In practice, however, there are well-known problems associated with *k*-percent rules, such as the instability of monetary aggregates and the choice of the appropriate monetary aggregate.

The paper then investigates the existence of simple interest rate rules that could replicate the *k*-percent money growth rule. The following rule is analyzed: let policymakers announce an inflation target and then change the nominal interest rate according to the difference between the current inflation rate and the inflation target. Provided that the inflation target is fully credible, such a policy generates the same qualitative dynamic adjustment as the one delivered by the *k*-percent money rule. In fact, a simple case is found (when real money demand is of the Cagan type and thus has a constant interest rate semielasticity) in which the inflation target-cum-interest rate rule exactly replicates the *k*-percent money growth rule. In other words, the adjustment of the economy to the announcement of a lower inflation target exactly replicates the response of the economy to a lower rate of monetary expansion. The paper then analyzes the following real interest rate rule: let policymakers announce an inflation target and change the real interest rate according to the difference between (a) the current inflation rate and the inflation target and (b) deviations of output from its full-employment level. Under a constant semielastic money demand, this rules also replicates the *k*-percent money growth rule.

These three rules—the *k*-percent money rule, an inflation target-cum-nominal interest rate rule, and an inflation target-cum-real interest rate rule—are identical for a Cagan money demand. However, as the analysis moves away from the more traditional instrument (a monetary aggregate) toward less traditional ones (a nominal interest rate and then a real interest rate), the feedback mechanisms multiply and the policy regime becomes more complicated. This should prove a useful conceptual benchmark. In practice, this increased level of complexity would need to be weighted against whatever practical advantages may exist. For instance, setting a short-term nominal interest rate may be operationally easier than setting a constant growth of the money supply.

The three equivalent policy rules just mentioned imply an adjustment toward a lower long-run inflation rate that involves a prolonged period of deflation (defined as inflation falling below its long-run level during the adjustment process). This is needed for real money balances to achieve a higher steady-state level. In practice, policymakers may want to avoid this deflationary period since it essentially implies that the monetary policy stance is too tight. As the analysis shows, policy rules that respond to the output gap may prevent this deflationary period and ensure a monotonic fall of the inflation rate toward its lower steady-state value. Under certain conditions, these three policy rules deliver exactly the same outcome: an inflation target combined with a money growth rule that responds to the output gap; an inflation target combined with a nominal interest rate rule that responds to both the inflation gap and the output gap; and an inflation target combined with a real interest rate rule that responds to both the inflation gap and the output gap. It is still the case that the money growth rule is the simplest of the three rules. But, again, a nominal interest rate rule can achieve the same results without adding too much complexity (since policymakers can easily monitor the inflation gap).

A final word of caution before proceeding to the formal analysis. The equivalence results derived in this paper provide a useful conceptual benchmark. No generality is claimed, however. Quite to the contrary, the whole point of the exercise is to find conditions (however strong) under which different policy rules are equivalent. As is typical of applied economic theory (think, for example, of the Ricardian equivalence or the Modigliani-Miller theorem), the idea behind this analysis is that understanding admittedly extreme cases in which such equivalences hold should then make it easier to identify the main factors explaining deviations from the benchmark in the real world. In this context, a model in which some basic policy equivalence results hold should provide a useful starting point for thinking about different monetary policy rules and instruments. Put differently, the idea is to provide a useful conceptual benchmark for thinking about these issues, rather than to construct a model that replicates the real world.

The paper proceeds as follows. Section 1 develops the basic model. Sections 2, 3, and 4 analyze the use of the money supply, the nominal interest rate, and the real interest rate as policy instruments, respectively. Section 5 derives the first equivalence proposition. Section 6 analyzes policy rules aimed at avoiding excessively tight monetary policy and derives the second equivalence proposition. Section 7 concludes.

1. The Model

Consider a closed economy inhabited by a very large number of identical consumers. Agents have perfect foresight. The Fischer equation holds, so that $i_t = r_t + \pi_t$, where i_t is the nominal interest rate, r_t is the real interest rate, and π_t is the inflation rate.

1.1 Consumers

The lifetime utility of the representative consumer is given by

$$\int_0^\infty u\left(c_t\right)\exp\left(-\beta t\right)\mathrm{d}t \ , \tag{1}$$

where c_t denotes consumption, $\beta(> 0)$ is the subjective discount rate, and the function $u(\cdot)$ satisfies $v'(\cdot) > 0$ and $v''(\cdot) < 0$.

Consumers hold two assets: a bond (indexed to the price level and in zero net supply in the aggregate) and money. Let a_t denote the household's financial wealth in real terms. Hence,

$$a_t = b_t + m_t \ , \tag{2}$$

where b_t and m_t denote the real stocks of bonds and money, respectively. The bond earns a nominal return of i_t.

In this economy, trading is a costly activity in terms of resources. Consumers hold money in order to reduce transactions costs. Transactions costs are thus given by $v(m_t)$, where $v'(m) < 0$ and $v''(m) > 0$.[2]

The consumer's flow constraint is given by

$$\dot{a}_t = r_t a_t + y_t + \tau_t - c_t - i_t m_t - v\left(m_t\right) , \tag{3}$$

where y_t denotes the output of the good and τ_t are lump-sum transfers from the government.

2. See Dornbusch and Frenkel (1973). Since transactions costs do not depend on consumption, the derived real money demand will not depend on consumption, either.

The consumer chooses (c_t, m_t) for all $t \in [0, \infty)$ to maximize lifetime utility (equation 1), subject to equation 3, for given paths of r_t, y_t, τ_t, and i_t, and a given value of a_0. The first-order conditions for this standard optimal control program are the following:

$$u'(c_t) = \lambda_t \,, \tag{4}$$

$$-v'(m_t) = i_t \,, \quad \text{and} \tag{5}$$

$$\dot{\lambda}_t = \lambda_t (\beta - r_t) \,, \tag{6}$$

where λ_t is the current value multiplier associated with constraint 3. Equation 4 indicates that, at an optimum, the household equates the marginal utility of consumption to the marginal utility of wealth. Equation 6 is the law of motion of the multiplier. Condition 5 states that, at an optimum, the benefits derived from holding an additional unit of real money balances will be equal to the corresponding opportunity cost. This equation implicitly defines a money demand function:

$$m_t = L(i_t) \,, \tag{7}$$

where $L'(i_t) = -\dfrac{1}{v''(m_t)} < 0$.

Differentiating equation 4 with respect to time and combining it with equation 6 leads to the familiar Euler equation:

$$\dot{c}_t = \frac{u'(c_t)}{-u''(c_t)} (r_t - \beta) \,. \tag{8}$$

Hence, if the real interest rate is above the rate of time preference, today's consumption is expensive relative to tomorrow's, and so consumption will increase over time.

1.2 Government

The government plays no active role. It gives back to consumers the proceeds from money creation and transactions costs, which are paid out as lump-sum transfers. The government's constraint is thus

$$\tau_t = \mu_t m_t + v(m_t) \,. \tag{9}$$

The fact that $v(m_t)$ appears in the government's flow constraint reflects the assumption that $v(m_t)$ is a private cost for consumers but not a social cost. Formally, one can think of some federal agency providing (at zero cost) the transactions costs needed by consumers. The profits of this federal agency are returned to households as lump-sum transfers. This assumption is made to eliminate wealth effects associated with changes in inflation, which would unnecessarily complicate the analysis.

1.3 Supply Side

Output is endogenous and assumed to be demand determined; that is, $y_t = c_t$. The inflation rate is assumed to be predetermined at each point in time. This formulation is meant to capture a situation in which widespread backward-looking indexation of prices and wages imparts a high degree of inertia to the inflation rate.[3] The change in the inflation rate is given by

$$\dot{\pi}_t = \gamma\left(\mu_t - \pi_t\right) + \alpha\left(c_t - \bar{y}\right), \tag{10}$$

where $\bar{y}$ denotes the full-employment level of output. Equation 10 says that the inflation rate will increase whenever it is below the rate of money growth, μ_t, or whenever aggregate demand exceeds full-employment output.

1.4 Equilibrium Conditions

Since bonds are so-called inside money in this economy, aggregate bond holdings must be zero:

$$b_t = 0 \; . \tag{11}$$

Of course, substituting this equation and goods market equilibrium, $y_t = c_t$, into the consumer's flow constraint (equation 3) yields the government's budget constraint (equation 9), which is simply a manifestation of Walras Law.

Finally, and for further reference, since $m = M/P$ by definition, it follows that

$$\dot{m}_t = m_t\left(\mu_t - \pi_t\right). \tag{12}$$

3. Widespread indexation has long characterized countries with chronic inflation. See, for instance, Dornbusch and Simonsen (1987); Edwards (1991); Bruno (1993).

2. A Fixed Money Growth Rule

As a benchmark, consider the case in which policymakers set the initial level and the growth rate of the nominal money supply (denoted by $\bar{\mu}$). Hence, equations 10 and 12 become

$$\dot{\pi}_t = \gamma\left(\bar{\mu} - \pi_t\right) + \alpha\left(c_t - \bar{y}\right) \quad \text{and} \tag{13}$$

$$\dot{m}_t = m_t\left(\bar{\mu} - \pi_t\right). \tag{14}$$

Substituting the Fisher equation and equation 5 into equation 8 produces

$$\dot{c}_t = \frac{u'\left(c_t\right)}{u''\left(c_t\right)}\left[\beta + v'\left(m_t\right) + \pi_t\right]. \tag{15}$$

Equations 13, 14, and 15 constitute a differential equation system in π_t, m_t, and c_t, for a given value of the policy variable $\bar{\mu}$. Both π_t and m_t are predetermined variables.

The system's steady state is given by

$$\pi_{ss} = \bar{\mu} \tag{16}$$

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{m}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} -\gamma & 0 & \alpha \\ -m_{ss} & 0 & 0 \\ u'\left(\bar{y}\right)/u''\left(\bar{y}\right) & \left[u'\left(\bar{y}\right)v''\left(m_{ss}\right)\right]/u''\left(\bar{y}\right) & 0 \end{bmatrix} \begin{bmatrix} \pi_t - \bar{\mu} \\ m_t - m_{ss} \\ c_t - \bar{y} \end{bmatrix}$$

$$c_{ss} = \bar{y}, \quad \text{and} \tag{17}$$

$$-v'\left(m_{ss}\right) = \beta + \bar{\mu}. \tag{18}$$

Linearizing this system around the steady state yields

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The trace and determinant of the matrix associated with the linear approximation are, respectively,

$$\text{Tr} = -\gamma < 0 \quad \text{and} \tag{19}$$

$$\Delta = -\alpha\, m_{ss} \frac{u'(\bar{y})v''(m_{ss})}{u''(\bar{y})} > 0 \ , \tag{20}$$

which implies that there is one positive root and two roots with negative real part.[4] Since there are two predetermined variables, the system exhibits saddle-path stability: for given initial values of π and m, c will adjust so as to position the system along its unique, perfect-foresight equilibrium path.

Let δ_i, i=1, 2, denote the two negative roots, with $\delta_1 > \delta_2$. Let $\mathbf{h}_{ij}$, j = 1, 2, 3, denote the elements of the eigenvector associated with root δ_i. For i = 1, 2, it follows that

$$\begin{bmatrix} -\gamma-\delta_i & 0 & \alpha \\ -m_{ss} & -\delta_i & 0 \\ u'(\bar{y})/u''(\bar{y}) & \left[u'(\bar{y})v''(m_{ss})\right]/u''(\bar{y}) & -\delta_i \end{bmatrix} \begin{bmatrix} \mathbf{h}_{i1} \\ \mathbf{h}_{i2} \\ \mathbf{h}_{i3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} . \tag{21}$$

Therefore,

$$\frac{\mathbf{h}_{i1}}{\mathbf{h}_{i2}} = -\frac{\delta_i}{m_{ss}} > 0 \ . \tag{22}$$

As becomes clear below, this provides a crucial piece of information when it comes to deriving the dynamic behavior of the system.

Setting to zero the constant corresponding to the unstable root, the solution to this dynamic system takes the form

$$\pi_t - \bar{\mu} = w_1 \mathbf{h}_{11} \exp(\delta_1 t) + w_2 \mathbf{h}_{21} \exp(\delta_2 t) \ , \tag{23}$$

$$m_t - m_{ss} = w_1 \mathbf{h}_{12} \exp(\delta_1 t) + w_2 \mathbf{h}_{22} \exp(\delta_2 t) \ , \quad \text{and} \tag{24}$$

$$c_t - \bar{y} = w_1 \mathbf{h}_{13} \exp(\delta_1 t) + w_2 \mathbf{h}_{23} \exp(\delta_2 t) \ , \tag{25}$$

4. In what follows, and to simplify the exposition, it is assumed that the roots with negative real part are real numbers. It can be checked that roots will be real (complex) numbers when γ is large (small) relative to α. This makes intuitive sense, because as can be seen from equation 13, a relatively large γ ensures that the rate of change of the inflation rate is relatively more responsive to the inflation rate differential.

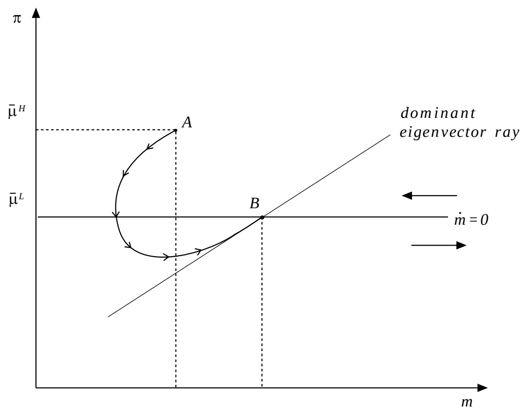
where w_i, i = 1, 2, denote the constants associated with root δ_i. Since $\delta_2 - \delta_1 < 0$, it follows that

$$\lim_{t\to\infty} \frac{\pi_t - \bar{\mu}}{m_t - m_{ss}} = \frac{\mathbf{h}_{11}}{\mathbf{h}_{12}} > 0 \,.$$

This implies that as t becomes large, inflation and real money balances will converge to their steady-state values from the same direction. Put differently, the dominant eigenvector ray, which is illustrated in figure 1, is positively sloped (see Calvo, 1987). Graphically, the system must converge asymptotically to the dominant eigenvector ray. Equation 14 also indicates that when $\pi_t > \bar{\mu}$ (or when $\pi_t < \bar{\mu}$), real money balances are falling (or rising). The corresponding directional arrows are drawn in figure 1.

We now have all the elements needed to study how this economy adjusts to an unanticipated and permanent fall in the monetary growth rate. Suppose that in the initial steady state (that is, for t < 0), the monetary growth rate is $\bar{\mu}^H$. At t = 0, policymakers announce an unanticipated and permanent reduction of the money growth rate from $\bar{\mu}^H$ to $\bar{\mu}^L$, where $\bar{\mu}^H > \bar{\mu}^L$. In terms of figure 1, the initial high-inflation steady state is at point A. The new steady state—with lower inflation and higher real money balances—is at point B. Given the conditions that must be satisfied by a convergent path, the economy must follow the arrowed path illustrated in figure 1.[5]

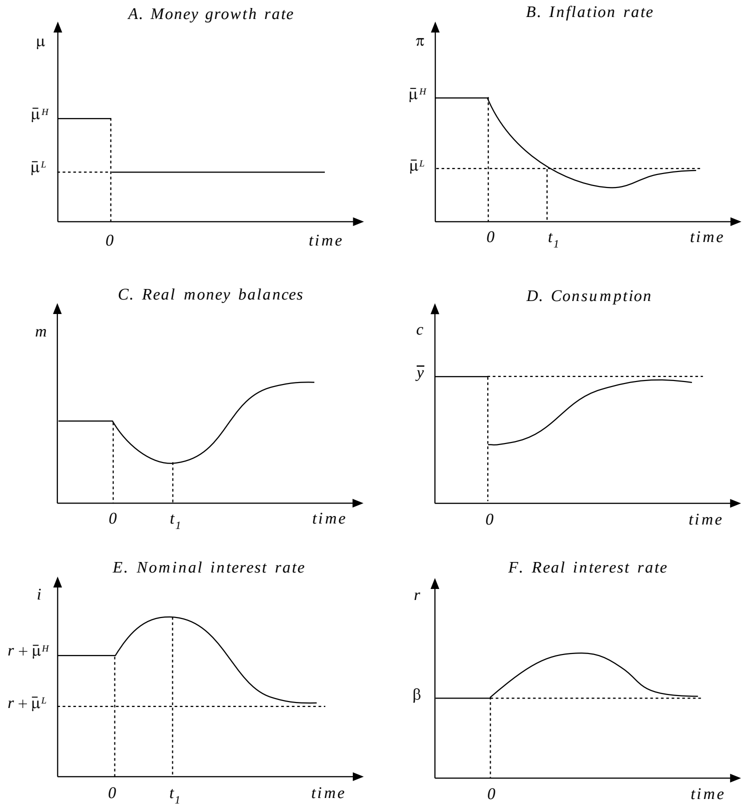
Figure 1. Dynamics in the (m, π) Plane



5. It can be ruled out that starting from point A, the system will first head in a northwestern direction. In other words, $\dot{\pi}_0 < 0$. To show this, one first must show that consumption jumps downwards on impact (see the appendix).

Figure 2 illustrates the time path of the main variables. The paths of inflation and real money balances follow directly from figure 1. Since the inflation rate is a predetermined variable, the reduction in the rate of monetary growth implies that real money balances fall in the early stages. Given the path of m_t, the path of i_t follows from equation 5. The nominal interest rate must rise early on to accommodate the lower level of real money balances. Since $r_t = i_t - \pi_t$, the path of r_t follows from the paths of i_t and π_t.[6] The fact that r_t is above β during the entire adjustment process implies that after jumping downward on impact, consumption rises throughout.

Figure 2. Reduction in Money Growth Rate



6. In principle, the possibility that the slope of the path of r_t changes sign more than once cannot be ruled out. What is important, however, is that during the adjustment process, r_t will always be above its unique steady-state value.

The intuition behind these results is as follows. The permanent reduction in the monetary growth rate implies that, in the new steady state, inflation—and thus the nominal interest rate—will be lower. Hence, real money demand in the new steady state will be higher. How will this increase in real money balances come about? Since the nominal money stock does not jump at $t = 0$ (it is a policy variable), the only way for the economy to generate higher real money balances is for the inflation rate to fall below the lower rate of monetary growth, $\overline{\mu}^L$. Inflation thus needs to undershoot its long-run value in order for real money balances to eventually begin to rise toward their higher steady-state value. In other words, tight monetary policy (in the form of a sharp reduction in the monetary growth rate) forces the economy to undergo a deflationary period. This tight monetary policy manifests itself in high nominal and real interest rates in the early stages of the stabilization program.

It will prove useful to derive analytically the time paths of the nominal and real interest rates. Differentiating first-order condition 5 and recalling that $\dot{m}_t / m_t = \overline{\mu} - \pi_t$, it follows that

$$\dot{i}_t = m_t v''(m_t)(\pi_t - \overline{\mu}^L). \qquad (26)$$

Along the adjustment path, the nominal interest rate thus depends on the difference between the current inflation rate and the long-run inflation rate (which equals $\overline{\mu}^L$). This already suggests that an inflation target combined with an interest rate rule whereby the nominal interest rate is raised if the actual inflation rate is above the inflation target may yield a similar dynamic path to the *k*-percent money growth rule.

With regard to the real interest rate, the Fischer equation and equations 13 and 26 can be used to obtain

$$\dot{r}_t = [\gamma + m_t v''(m_t)](\pi_t - \overline{\mu}^L) - \alpha(c_t - \overline{y}). \qquad (27)$$

Hence, the real interest rate increases whenever inflation is above its long-run value and falls if consumption is above the full-employment level of output. The intuition for the latter channel is as follows: other things being equal, when consumption is above its full-employment level, the inflation rate is rising (recall equation 13), which implies that the real interest rate is falling. Again, this suggests that an inflation target combined with a real interest rate rule that responds to both the output gap and the gap between the current inflation rate and an inflation target should yield similar dynamics to the *k*-percent money growth rule.

3. The Nominal Interest Rate as an Instrument

3.1 A Pure Interest Rate Peg

Suppose that policymakers set the nominal interest rate at a constant level, $\bar{i}$. This is achieved by letting the money supply adjust to whatever level is needed for the targeted interest rate to prevail. It will be shown that an interest rate peg leads to a multiplicity of equilibrium paths.

If $i_t=\bar{i}$, it follows from first-order condition 5 that $\dot{m}=0$ and, therefore, that $\pi_t=\mu_t$. Equation 10 then becomes

$$\dot{\pi}_t = \alpha\left(c_t - \bar{y}\right). \tag{28}$$

Taking into account the Fisher equation, the Euler equation 8 can be rewritten as

$$\dot{c}_t = \frac{u'\left(c_t\right)}{u''\left(c_t\right)}\left(\beta + \pi_t - \bar{i}\right). \tag{29}$$

Equations 28 and 29 constitute a differential equation system in π_t and c_t, for a given value of $\bar{i}$. At the steady state, $c_{ss}=\bar{y}$ and $\pi_{ss}=\bar{i}-\beta$. Linearizing the system around the steady state,

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} 0 & \alpha \\ u'\left(\bar{y}\right)/u''\left(\bar{y}\right) & 0 \end{bmatrix} \begin{bmatrix} \pi_t - \pi_{ss} \\ c_t - \bar{y} \end{bmatrix}.$$

The trace and determinant of the matrix associated with the linear approximation are given by, respectively,

$$\mathrm{Tr}=0 \quad \text{and}$$

$$\Delta = -\alpha \frac{u'\left(\bar{y}\right)}{u''\left(\bar{y}\right)} > 0.$$

This implies that there are two complex roots with real part equal to zero.[7] The dynamic system exhibits a vortex and is thus unstable. For a given initial value of the inflation rate (except if it happens to be the steady-state value), there is no value of consumption that places the economy on a convergent path. All possible values imply that the system will oscillate forever without ever reaching the steady state. In this model, therefore, a pure nominal interest rate peg does not provide a sensible way of conducting monetary policy.

Intuitively, the problem lies in the fact that under a pure interest rate peg, the economy loses its nominal anchor as the rate of monetary growth passively accommodates inflation. Indeed, as made clear by equation 28, there is no long-run value of the inflation rate (that is, no nominal anchor) to guide the inflation rate to a specific value. In contrast, under a *k*-percent money rule (recall equation 13), the rate of change of the inflation rate is affected by the difference between the current inflation rate and its long-run value.

3.2 A Nominal Interest Rate Rule with an Inflation Target

Consider the following policy regime: policymakers announce an inflation target, $\overline{\pi}$, and follow the interest rate rule

$$\dot{i} = \theta\left(\pi_t - \overline{\pi}\right), \qquad (30)$$

whereby the nominal interest rate is gradually raised (reduced) whenever the inflation rate is above its target.[8] (In this setup, the nominal interest rate is, by construction, a predetermined variable.) Differentiating first-order condition 5 and using equation 30 produces

$$\dot{m}_t = \frac{-\theta}{v''\left(m_t\right)}\left(\pi_t - \overline{\pi}\right). \qquad (31)$$

7. The roots are given by $\delta_{1,2} = \pm z\sqrt{-\alpha\left[u'\left(\overline{y}\right)/u''\left(\overline{y}\right)\right]}$, where $z^2 = -1$.
8. All policy parameters are positive unless otherwise indicated.

Combining equation 31 with equations 10 and 12 yields

$$\dot{\pi}_t = \frac{\gamma\,\theta}{m_t v''(m_t)}\left(\overline{\pi} - \pi_t\right) + \alpha\left(c_t - \overline{y}\right) . \tag{32}$$

This last equation, together with equations 15 and 31, forms a dynamic system in π_t, m_t, and c_t whose dynamic properties are qualitatively the same as those of the system described by equations 13, 14, and 15 for the fixed money growth rule. Thus, if at time 0, policymakers announce a reduction in the inflation target and follow policy rule 30, the economy will follow (in qualitative terms) the adjustment process depicted in figure 2, except for μ_t.

Note that μ_t is now an endogenous variable whose path is given by

$$\mu_t = \frac{\theta}{m_t v''(m_t)}\overline{\pi} + \left[1 - \frac{\theta}{m_t v''(m_t)}\right]\pi_t . \tag{33}$$

To fix ideas, note that $m_t v''(m_t)$ is the inverse of the absolute value of the semielasticity of real money demand (denoted by η^s); that is,

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If real money demand is of the Cagan type (that is, it exhibits a constant semielasticity), then it follows from equation 33 that on impact m_t will fall by more than the inflation target if $\eta^s > 1/\theta$ and by less if $\eta^s < 1/\theta$. In the case in which $\eta^s = 1/\theta$, $\mu_t = \overline{\pi}$ for all t, and the system behaves exactly as it does under a fixed money growth rule. In other words, an outside observer would not be able to tell whether a given reduction in the long-run inflation rate was brought about by a permanent reduction in $\overline{\mu}$ or by the announcement of a lower inflation target together with interest rate rule 30.

4. The Real Interest Rate as Policy Instrument

4.1 A Pure Real Interest Rate Peg

Suppose that policymakers set the real interest rate at the constant value . A necessary condition for such a real interest rate peg to be consistent with a convergent equilibrium path is that $\bar{r}=\beta$. Otherwise, consumption would either increase or decrease forever, as follows from equation 8. The peg $\bar{r}=\beta$ therefore implies that $\dot{c}_t = 0$. Hence, $c_t = \bar{y}$ for a convergent path to exist. From equation 10,

$$\dot{\pi}_t = \gamma\left(\mu_t - \pi_t\right). \tag{34}$$

Recalling the Fisher equation, it then follows from equations 7 and 34 that

$$\dot{\pi}_t\left[\frac{1}{\gamma} - \frac{L'\left(i_t\right)}{L\left(i_t\right)}\right] = 0 ,$$

which implies that along a perfect-foresight equilibrium path, $\dot{\pi}_t = 0$. Intuitively, if the monetary growth rate were above the inflation rate, real money balances would be increasing over time. The nominal interest rate would need to fall over time for money market equilibrium to hold. Since the real interest rate is constant, this implies that the inflation rate would be falling over time. In contrast, if the monetary growth rate were above the inflation rate, equation 34 indicates that inflation must be increasing over time. The only consistent path is for inflation to remain flat over time.

Let $\pi_t = \bar{\pi}$. Along a perfect-foresight equilibrium path, $\mu_t = \bar{\pi}$, $i_t = \bar{r} + \bar{\pi}$, and $m_t = L(\bar{r} + \bar{\pi})$. The economy is thus always in a stationary equilibrium. This equilibrium is not uniquely determined, however. To see this, suppose that for whatever reason, the public came to expect that the inflation rate will be $2\bar{\pi}$. By the above reasoning, $\mu_t = 2\bar{\pi}$ and $i_t = \bar{r} + 2\bar{\pi}$. Since the nominal interest rate is higher, real money demand would be lower; that is, $m_t = L(\bar{r} + 2\bar{\pi})$. The nominal money stock would fall to accommodate the lower real money demand. In sum, policymakers will validate any inflation rate expected by the public. There is nothing to tie down the level of the constant rate of inflation.

4.2 A Real Interest Rate Rule with an Inflation Target

Consider the following policy regime: policymakers announce an inflation target, $\bar{\pi}$, and follow the real interest rate rule

$$\dot{r}_t = \theta\left(\pi_t - \bar{\pi}\right). \tag{35}$$

Combining equations 7, 10, and 35 generates

$$\dot{\pi}_t = \frac{\theta\gamma\left[L'(i_t)/L(i_t)\right]}{1-\gamma\left[L'(i_t)/L(i_t)\right]}\left(\pi_t - \overline{\pi}\right) + \frac{\alpha}{1-\gamma\left[L'(i_t)/L(i_t)\right]}\left(c_t - \overline{y}\right).$$

Equations 8, 35, and 36 constitute a dynamic system in π_t, c_t, and r_t. Proceeding as in the case of the fixed money growth rule, this dynamic system may be solved by computing the dominant eigenvector ray. In response to an unanticipated reduction in the inflation target, the system adjusts to the new steady state following similar dynamics as in figure 2, except for the behavior of μ_t, which may vary over time.

5. An Equivalence Proposition

The above analysis has shown that both a nominal interest rate rule and a real interest rate rule, in conjunction with an inflation target, will qualitatively yield the same results as a *k*-percent money growth rule. I now discuss a particular case in which these three rules are exactly the same.

Suppose that the transactions cost technology takes the form

$$, \qquad (37)$$

where χ is a positive parameter.
Using equation 5, the real money demand then becomes

$$m_t = e^{\sigma(\chi - i_t) - 1}. \qquad (38)$$

This is a Cagan-type real money demand since it exhibits a constant semielasticity:

$$\eta^s(m_t) \equiv \frac{-\partial L(i_t)}{\partial i_t}\frac{1}{m_t} = \frac{1}{m_t v''(m_t)} = \sigma. \qquad (39)$$

Under such a specification, the paths for the nominal interest rate and the real interest rate for the *k*-percent money growth rule studied above are given, respectively, by

$$\dot{i}_t = \frac{1}{\sigma}\left(\pi_t - \bar{\mu}\right) \quad \text{and} \tag{40}$$

$$\dot{r}_t = \left(\gamma + \frac{1}{\sigma}\right)\left(\pi_t - \bar{\mu}\right) - \alpha\left(c_t - \bar{y}\right), \tag{41}$$

as follows from equations 26, 27, and 39. Equation 40 indicates that under a Cagan money demand, the rate of change of the nominal interest rate is a linear function of the gap between the current inflation rate and the long-run inflation rate (given by $\bar{\mu}$). From equation 41, it follows that the rate of change of the real interest rate is also a linear function of the inflation gap and, in addition, of the gap between consumption (aggregate demand) and full-employment output. Recall also from figure 2 that neither the nominal interest rate nor the real interest rate jump on impact (that is, at $t = 0$).

Suppose now that policymakers set an inflation target, $\bar{\pi}$, that pins down the long-run inflation rate and therefore plays the role of $\bar{\mu}$. In conjunction with this inflation target, they follow interest rate rules of the form in equations 40 and 41, whereby the nominal interest rate and the real interest rate are changed gradually over time in response to changes in either the inflation gap or the output gap. Furthermore, suppose that policymakers set the policy reaction coefficients equal to $1/\sigma$ for the inflation gap in the case of the nominal interest rate rule and equal to $\gamma + 1/\sigma$ for the inflation gap and $-\alpha$ for the output gap in the case of the real interest rate rule. Under these conditions, it follows that these two policy rules will be exactly equivalent to the fixed money growth rule studied above.

This equivalence result can be summarized in the following proposition:

Proposition 1: If the transactions technology is given by equation 37, which gives rise to a Cagan money demand function (equation 38) with semielasticity equal to σ, then the following three monetary policy rules are exactly equivalent:

(a) policymakers set a fixed money growth rule,

$$u_t = \overline{\mu}\ ; \tag{42}$$

(b) policymakers announce an inflation target, $\overline{\pi}$ (equal to $\overline{\mu}$), and follow a nominal interest rate rule,

$$\dot{i}_t = \theta\left(\pi_t - \overline{\mu}\right), \tag{43}$$

where $\theta = 1/\sigma$; and

(c) policymakers announce an inflation target, $\overline{\pi}$ (equal to $\overline{\mu}$), and follow a real interest rate rule,

$$\dot{r}_t = \theta^1\left(\pi_t - \overline{\mu}\right) + \theta^2\left(c_t - \overline{y}\right), \tag{44}$$

where $\theta^1 = \gamma + 1/\sigma$ and $\theta^2 = -\alpha$.

This policy equivalence implies that if policymakers wish to reduce inflation, any of the three policy regimes are formally equivalent. Assuming that the model is a reasonably good description of reality, the choice between different rules will come down to practical advantages. Several remarks are in order.

First, as one moves from the more orthodox instrument (the money supply) to the less orthodox instrument (the real interest rate), the rules become more complicated in the sense that more feedback mechanisms are required. Hence, in practice, these policy rules will be increasingly complex to implement.

Second, not only is the real interest rate rule the more complicated (as it depends on the output gap, which is clearly difficult to estimate in practice), but it also does not have, in principle, any advantage over the nominal interest rate. In addition, the real interest rate rule requires that the real interest rate be reduced when there is excess aggregate demand, which is the opposite of what the public may think should be done.[9] This might lead to credibility problems.

Finally, in practice, monetary authorities have mostly abandoned money growth rules based principally on the instability of money demand and the problems associated with choosing between

9. Of course, a real interest rate rule of the form in equation 35—which does not respond to the output gap—can still deliver a qualitatively similar adjustment to the fixed money growth rule, although it cannot replicate it exactly. Even in this case, however, the real interest rate does not have any advantages over the nominal interest rate as a policy instrument.

different monetary aggregates. A nominal interest rate rule avoids this problem and therefore might be preferable. This might explain the increasing popularity of inflation targeting regimes, of which rule 30 may be considered a particular case.

6. Avoiding Deflation: Alternative Rules

A legitimate question that may arise is why the fixed money growth rule should be the benchmark against which other rules are compared. Two factors make it the natural benchmark. First, it is the traditional policy instrument par excellence. Second, as shown above, it is the only instrument that can be set with no feedback rules. In the absence of feedback rules, neither the nominal nor the real interest rate can provide a nominal anchor for monetary policy.

A case might still be made for considering other possible benchmarks. The main rationale for doing so is the fact that, as discussed above, a fixed money growth rule represents an excessively tight monetary policy stance, as it requires a prolonged period of deflation (in the sense of the inflation rate falling below its long-run value) in order to build real money balances (see figure 2, panel B). One may therefore wonder whether other rules could avoid this deflationary period and thus provide a better benchmark. Since the excessively tight monetary policy is reflected in an initial fall in consumption, it seems natural to ask whether an interest rate rule that also responds to the output gap would be capable of avoiding the deflationary period.[10]

6.1 A Nominal Interest Rate Rule with Output Feedback

Suppose that instead of equation 30, the interest rate rule takes the form:

$$\dot{i} = \theta\left(\pi_t - \overline{\pi}\right) + \xi\left(c_t - \overline{y}\right). \tag{45}$$

This rule captures policymakers' concerns about consumption (and thus output) falling below its full-employment level. If consumption is below its full-employment level, the nominal interest rate is reduced.

10. For simplicity, I consider the case in which real money demand is given by equation 38.

Using equations 5 and 10, together with the fact that $\dot{m}_t / m_t = \mu_t - \pi_t$, yields,

$$\dot{\pi}_t = \gamma\theta\eta^s\left(\overline{\pi} - \pi_t\right) + \left(\alpha - \gamma\xi\eta^s\right)\left(c_t - \overline{y}\right). \tag{46}$$

Since the main purpose of the exercise is to provide an example in which a nominal interest rate with output feedback avoids the deflationary period, the analysis considers the case in which $\alpha - \gamma\xi\eta^s = 0$. Equation 46 then simplifies to:

$$\dot{\pi}_t = \gamma\theta\eta^s\left(\overline{\pi} - \pi_t\right). \tag{47}$$

This is a stable differential equation in π_t. It follows immediately that an unanticipated and permanent reduction in the inflation target combined with rule 45 will cause inflation to fall monotonically over time toward its lower steady-state value.

To solve for the whole dynamic system, combine the Fisher equation and equations 45 and 47. This generates the law of motion for the real interest rate:

$$\dot{r}_t = \theta\left(1 + \gamma\eta^s\right)\left(\pi_t - \overline{\pi}\right) + \xi\left(c_t - \overline{y}\right). \tag{48}$$

Unlike in the *k*-percent money growth rule case in which $\dot{r}_t$ depended negatively on excess demand (recall equation 41), here it depends positively. The intuition in this case is as follows. Excess demand does not directly affect the change in the inflation rate, as follows from equation 47. Other things being equal, excess aggregate demand therefore leads to an increase in the nominal interest rate—according to rule 45—and thus in the real interest rate.

Equations 8, 47, and 48 form a differential equation system in π_t, r_t, and c_t. Both π_t and r_t are predetermined variables.[11] Linearizing this system around the steady state,

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{r}_t \\ \dot{c}_t \end{bmatrix} = \begin{bmatrix} -\gamma\theta\eta^s & 0 & 0 \\ \theta\left(1 + \gamma\eta^s\right) & 0 & \xi \\ 0 & -u'\left(\overline{y}\right)/u''\left(\overline{y}\right) & 0 \end{bmatrix} \begin{bmatrix} \pi_t - \overline{\pi} \\ r_t - \beta \\ c_t - \overline{y} \end{bmatrix}.$$

11. By construction i_t is predetermined. Since π_t is also a predetermined variable, so is r_t.

The trace and determinant of the matrix associated with the linear approximation are, respectively,

$$\mathrm{Tr} = -\gamma\theta\eta^s < 0 \quad \text{and}$$

$$\Delta = \frac{u'(\bar{y})}{-u''(\bar{y})}\xi\gamma\theta\eta^s > 0 .$$

The system thus has two negative roots.[12] Let δ_i, i = 1, 2, denote the two negative roots.[13] Let $\mathbf{h}_{ij}$, j = 1, 2, 3, denote the elements of the eigenvector associated with root δ_i. For i = 1, 2, it follows that

$$\begin{bmatrix} -\gamma\theta\eta^s - \delta_i & 0 & 0 \\ \theta\left(1+\gamma\eta^s\right) & -\delta_i & \xi \\ 0 & -u'(\bar{y})/u''(\bar{y}) & -\delta_i \end{bmatrix} \begin{bmatrix} \mathbf{h}_{i1} \\ \mathbf{h}_{i2} \\ \mathbf{h}_{i3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} .$$

Therefore,

$$\frac{\mathbf{h}_{i2}}{\mathbf{h}_{i3}} = -\frac{\delta_i}{-u'(\bar{y})/u''(\bar{y})} < 0 \cdot$$

Setting to zero the constant corresponding to the unstable root, the solution to this dynamic system takes the form

$$\pi_t - \bar{\pi} = w_1\mathbf{h}_{11}\exp(\delta_1 t),$$

$$r_t - \beta = w_1\mathbf{h}_{12}\exp(\delta_1 t) + w_2\mathbf{h}_{22}\exp(\delta_2 t), \quad \text{and}$$

$$c_t - \bar{y} = w_1\mathbf{h}_{13}\exp(\delta_1 t) + w_2\mathbf{h}_{23}\exp(\delta_2 t),$$

where w_i, i = 1, 2, denote the constants associated with root δ_i. (Note that $\mathbf{h}_{21} = 0$.) Since $\delta_1 - \delta_2 < 0$, it follows that

$$\lim_{t\to\infty}\frac{r_t - \beta}{c_t - \bar{y}} = \frac{\mathbf{h}_{12}}{\mathbf{h}_{13}} < 0 .$$

12. Since one of the negative roots is $-\gamma\theta\eta^s$, it follows that the two negative roots are real numbers.

13. The roots are $\delta_1 = -\gamma\theta\eta^s$ and $\delta_2 = -\sqrt{\xi\frac{u'(\bar{y})}{-u''(\bar{y})}}$. (It is assumed, with no loss of generality, that $\delta_1 < \delta_2$.)

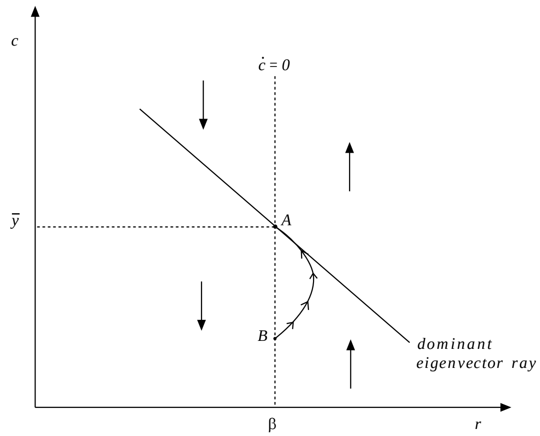
This implies that as t becomes large, the real interest rate and consumption will converge to their steady-state values from opposite directions. In other words, the dominant eigenvector ray is negatively sloped (figure 3). From equation 8, it follows that consumption increases (falls) to the right (left) of $r_t = \beta$. The corresponding directional arrows are drawn in figure 3.

Turning now to the economy's response to a reduction in the inflation target, $\overline{\pi}$, suppose that in the initial steady state (that is, for $t < 0$), the inflation target is $\overline{\pi}^H$. (The initial steady state is denoted by point A in figure 3.) At $t = 0$, policymakers announce an unanticipated and permanent reduction of the inflation target from $\overline{\pi}^H$ to $\overline{\pi}^L$, where $\overline{\pi}^H > \overline{\pi}^L$. In terms of figure 3, the steady state remains at point A. To be on a convergent path, c_t must jump down on impact to a point such as B.[14] The system then follows the arrowed path back to point A.

Figure 4 illustrates the time path of the main variables. If $\theta = 1/\eta^s$, the path of the monetary growth rate is given by

$$\mu_t = \overline{\pi}^L - \frac{\xi}{\theta}\left(c_t - \overline{y}\right). \tag{49}$$

Figure 3. Dynamics in the (*r*, *c*) Plane

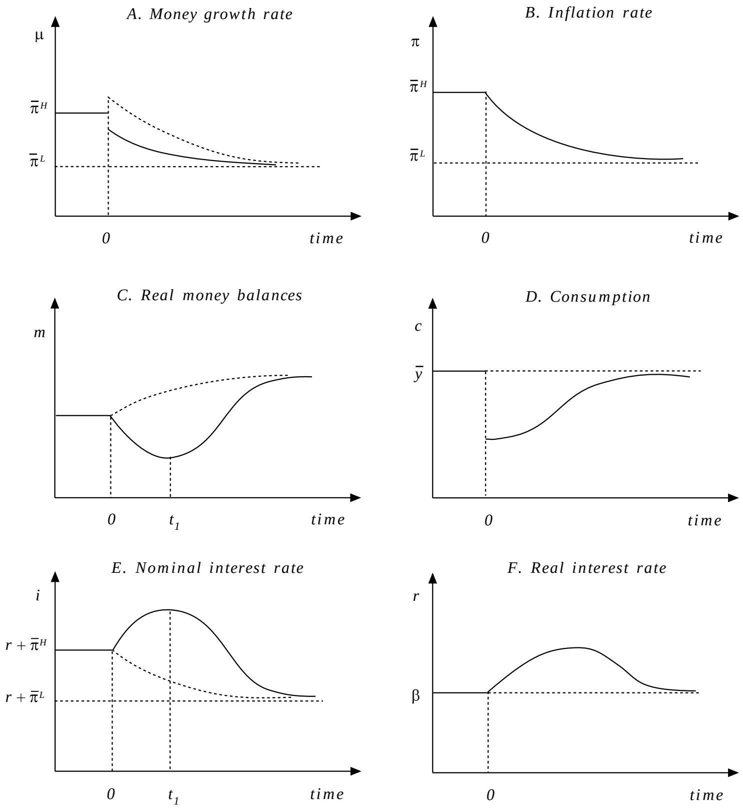


14. If consumption did not jump downwards on impact, the system would diverge in a northeastern direction. To see this notice that, if $c_0 \geq \overline{y}$, then $\dot{r}_0 = \theta\left(1 + \gamma\eta^s\right)\left(\pi_0 - \overline{\pi}\right) + \xi\left(c_0 - \overline{y}\right) > 0$. The rising real interest rate would in turn lead to a rising path of consumption.

Thus the money growth rate will fall on impact by less than the inflation target. During the transition, the money growth rate must be higher than the inflation rate (at least for some period of time) to allow real money balances to grow over time without the need for inflation to fall below its steady-state value. Real money balances fall in the initial stages and increase later on (panel C), while the nominal interest rate increases early on and falls later on (panel D).

Depending on the parameter configuration, the money growth rate could rise on impact (as captured by the dashed path in panel A). In that case, real money balances will grow from the very beginning and the nominal interest rate will fall throughout the adjustment path (dashed paths in panels C and E, respectively).

Figure 4. Reduction in Inflation Target



6.2 A Second Equivalence Proposition

It follows immediately from equation 49 that in conjunction with an inflation target, a money growth rule that responds to the output gap would enable policymakers to reduce inflation while avoiding a protracted deflationary period. Furthermore, a real interest rate rule that takes the form given by equation 48 would also avoid the deflationary period. A key feature of this rule is that now the real interest rate would be raised when the economy is overheated, whereas rule 44 implies the opposite.

This establishes another equivalence between the different policy rules and instruments, which is summarized in the following proposition:

Proposition 2: If the transactions cost technology is given by equation 37, which gives rise to a Cagan money demand function (equation 38) with semielasticity equal to σ, then the following three monetary policy rules are exactly equivalent (under the maintained assumption that $\theta = 1/\eta^s$):

(a) policymakers announce an inflation target, $\overline{\pi}$, and follow the money growth rule,

$$\mu_t = \overline{\pi} - \xi\sigma\left(c_t - \overline{y}\right);$$

(b) policymakers announce an inflation target, , and follow the nominal interest rate rule,

$$\dot{i} = \theta\left(\pi_t - \overline{\pi}\right) + \xi\left(c_t - \overline{y}\right), \quad \text{and}$$

(c) policymakers announce an inflation target, $\overline{\pi}$, and follow the real interest rule,

$$\dot{r}_t = \theta^1\left(\pi_t - \overline{\pi}\right) + \theta^2\left(c_t - \overline{y}\right),$$

where $\theta^1 = \theta\left(1 + \gamma\sigma\right)$ and $\theta^2 = \xi$.

It is still the case that the money rule is the simplest one. On the other hand, all rules depend on the output gap, which is naturally difficult to gauge in practice. Given the difficulties associated with money rules, therefore, the nominal interest rate-cum-inflation target policy regime continues to look like a very reasonable alternative to money rules.

7. Final Remarks

This paper has established some basic equivalences among alternative instruments and policy rules in the context of a closed-economy model with sticky inflation. It has shown that a long-run reduction in the inflation rate can be achieved with three different rules—a *k*-percent money growth rule, an interest rate rule, and a real interest rate rule—which deliver exactly the same outcome. The money rule is the simplest, however, as it involves no feedback mechanisms. If policymakers wish to avoid a protracted deflationary period, there are also three different policy regimes that deliver exactly the same outcome.

The goal of the analysis has been to put enough structure into the model so as to establish some basic policy equivalences, which should be helpful in thinking about alternative policy regimes. The main policy conclusion of the analysis is perhaps that a nominal interest rate rule combined with an inflation target can, in principle, replicate exactly the workings of a money growth rule. Taken as a normative result, this provides strong support for using nominal interest rate rules, given the well-known practical difficulties of controlling monetary aggregates. It may explain the dramatic shift in actual policymaking away from monetary targets and toward regimes that essentially involve—implicitly or explicitly—an inflation target and a nominal interest rate rule aimed at achieving that target.

How would the main conclusions of the analysis be affected by relaxing some of the central assumptions? This is an area for future research, but some conjectures may be made. Consider the case of an open economy. In line with the spirit of the model, it could be assumed that the prices of tradables goods are flexible and determined by purchasing power parity, while inflation of home goods is sticky and determined in the same way as in the closed-economy model. Under flexible exchange rates, such a model should generate the same results. The reason is simply that under flexible exchange rates, the money supply remains the main nominal anchor; the same equivalences with interest rate rules would therefore hold. I would thus conjecture that, to a first approximation, the results should hold for flexible exchange rate regimes.

Under fixed or predetermined exchange rates, the nature of the policy rules studied in the paper would need to be modified to account for the fact that the nominal money supply is endogenous. This implies that the nominal interest rate (and real interest rate) could jump on impact, in contrast to this paper's model. One would thus need to study interest rate rules that could include an initial discrete change in interest rates.[15] This may complicate the formal analysis, but I see no reason to believe that it would alter the main message of this paper.

What if the economy were subject to stochastic shocks? While this is an extension worth addressing, conceptually it is not obvious why it should fundamentally alter any of the main conclusions. Suppose there were stochastic shocks to money demand. A *k*-percent money rule would absorb such shocks by variations in nominal (and thus real) interest rates. A nominal (or real) interest rate rule that responded to such shocks should deliver a similar outcome. If, on the other hand, certain types of shock were more prevalent than others, this might affect the choice of instruments along the lines of Poole (1970). It is unclear, however, how such considerations would affect the present analysis, since the type of indeterminacies emphasized by the more modern, rational-expectations literature do not depend on the specific shocks that hit the economy.

15. Of course, the same logic would apply to a closed-economy model in which the real money demand also depended on consumption (which is not the case in this model).

APPENDIX A

Fixed Money Growth Rule

A.1 Initial Jump in Consumption

Following an unanticipated reduction in the monetary growth rate, consumption must jump downward on impact. The proof proceeds in three stages.

First, different elements of the eigenvectors must be signed. With no loss of generality, let $\mathbf{h}_{11} = \mathbf{h}_{21} = 1$. From equation 22, it then follows that

$$\mathbf{h}_{12} = -\frac{m_{ss}}{\delta_1} > 0 \ , \tag{A1}$$

$$\mathbf{h}_{22} = -\frac{m_{ss}}{\delta_2} > 0 \ , \text{ and} \tag{A2}$$

$$\mathbf{h}_{22} - \mathbf{h}_{12} = m_{ss}\left(\frac{1}{\delta_1} - \frac{1}{\delta_2}\right) < 0 \ . \tag{A3}$$

(To sign the last expression, recall that, by construction, $\delta_1 - \delta_2 > 0$). From system 21, it follows that

$$\mathbf{h}_{i3} = \frac{\gamma + \delta_i}{\alpha} \quad , \tag{A4}$$

where i = 1, 2 .

Second, I solve for the system's constants. The solution to the dynamic system (given by equations 23, 24, and 25) leads to

$$\pi_0 - \overline{\mu}^L = \omega_1 + \omega_2 \ , \tag{A5}$$

$$m_0 - m_{ss} = \omega_1 \mathbf{h}_{12} + \omega_2 \mathbf{h}_{22} \ , \text{ and} \tag{A6}$$

$$c_0 - \overline{y} = \omega_1 \mathbf{h}_{13} + \omega_2 \mathbf{h}_{23} \ . \tag{A7}$$

Equations A5 and A6 can be used to solve for ω_1 and ω_2:

$$\omega_1 = \frac{\mathbf{h}_{22}\left(\pi_0 - \overline{\mu}^L\right) - \left(m_0 - m_{ss}\right)}{\mathbf{h}_{22} - \mathbf{h}_{12}} < 0 \quad \text{and} \tag{A8}$$

$$\omega_2 = \frac{m_0 - m_{ss} - \mathbf{h}_{12}\left(\pi_0 - \overline{\mu}^L\right)}{\mathbf{h}_{22} - \mathbf{h}_{12}} > 0\ , \tag{A9}$$

where the signs follow from equations A1, A2, and A3 and the fact that $\pi_0 - \overline{\mu}^L > 0$ and $m_0 - m_{ss} < 0$.

Finally, the expression for the initial jump in consumption can be signed. Substituting equations A4, A8, and A9 into equation A7 yields

$$c_0 - \overline{y} = \frac{1}{\alpha}\left[\frac{m_0 - m_{ss}}{m_{ss}}\,\delta_1\delta_2 + \left(\gamma + \delta_1 + \delta_2\right)\left(\pi_0 - \overline{\mu}^L\right)\right] < 0\ . \tag{A10}$$

To sign the last expression, note that $\gamma + \delta_1 + \delta_2 < 0$, which follows from the fact that $\delta_1 + \delta_2 + \delta_3 + \gamma = 0$ (recall, from equation 19, that the system's trace is $-\gamma$) and $\delta_3 > 0$. Equation A10 implies that $c_0 < \overline{y}$. Consumption thus jumps downward on impact (that is, at $t = 0$).

A.2 Sign of $\dot{\pi}_0$

To check that the system will not initially head in a northwestern direction starting from point A (in terms of figure 1), it is enough to show that $\dot{\pi}_0 < 0$. To that effect, equation 13 can be used to show that

$$\dot{\pi}_0 = \gamma\left(\overline{\mu}^L - \pi_0\right) + \alpha\left(c_0 - \overline{y}\right) < 0\ , \tag{A11}$$

since $\overline{\mu}^L - \pi_0 < 0$ and, as just shown, $c_0 - \overline{y} < 0$.

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